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نموذج إجابة مادة ضوء
د./ صلاح عيد ابراهيم حمزة

Part one "Optics"

Q1) -- [30 marks]

Derive an expression for the mirror equation.

Answer

We can us the geometry shown in Fig. (5) to calculate the image distance q from a knowledge of the object distance p and the mirror radius of curvature, R. By convention, these distances are measured from point V. We assume that the object at point O. Therefore, any ray leaving O is reflected at the spherical surface and focus at a point I, the image point. Let us proceed by considering the geometric construction in Fig. (5), which shows a single ray leaving point O and focusing at point I.

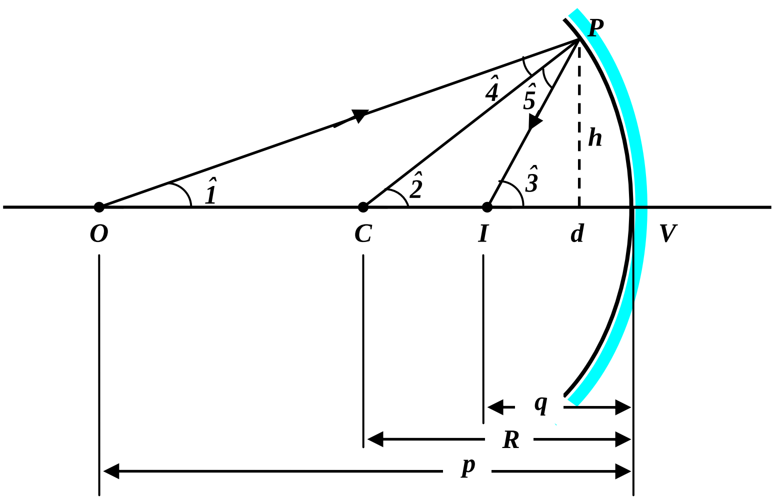

Fig. (5): The image formed by a spherical concave mirror where the object *O* lies outside the center of curvature, *C*.

We assume the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this to the triangles OPC and OPI gives:

$$\hat{2} = \hat{1} + \hat{4}, \tag{1}$$

$$\hat{3} = \hat{2} + \hat{5}. \tag{2}$$

From Eqs. (1) and (2)

$$\hat{4} = \hat{2} - \hat{1} \qquad \text{and} \qquad \hat{5} = \hat{3} - \hat{2}$$

From the law of reflection $\hat{4} = \hat{5}$. So

$$\hat{2} - \hat{1} = \hat{3} - \hat{2}$$

or

$$\hat{3} + \hat{1} = 2(\hat{2}). \tag{3}$$

From the triangles in Fig. (5), we note that:

$$\tan \hat{1} = \frac{h}{p-d}, \quad \tan \hat{3} = \frac{h}{q-d}, \quad \text{and} \quad \tan \hat{2} = \frac{h}{R-d}$$

The substitution in Eq. (3) gives

$$\frac{h}{p-d} + \frac{h}{q-d} = 2\left(\frac{h}{R-d}\right)$$

or

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{2}{R}} \tag{4}$$

This equation is called the ***mirror equation***. If the object is very far from the mirror, *p* can be said to approach infinity, then $\frac{1}{p} \approx 0$, and we see from Eq. (6) that $q \approx \frac{R}{2}$. That is, when the object is very far from the mirror, the image point is halfway between the center of the curvature and the center of the mirror, as in Fig. (6). The rays are essentially parallel in this figure and we call the image point in this special case the ***focal point***, *F*, and the image distance the ***focal length***, *f*, where

$$f = \frac{R}{2}. \tag{5}$$

The mirror equation can be expressed in terms of the focal length:

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.} \tag{6}$$

Q2) -- [30 marks]

Assume that a certain concave spherical mirror has a focal length of 10 cm. (a) Locate the image and find the magnification for an object distance of 25 cm. (b) Determine whether the image is real or virtual, inverted or upright.

Answer

(a) Find the image position for an object distance of 25.0 cm.

Calculate the magnification and describe the image.

Use the mirror equation to find the image distance:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{25}+\frac{1}{q}=\frac{1}{10}$$

$$q=16.7\,\text{cm}$$

Because *q* is positive, the image is in front of the mirror and is real. The magnification is given by

$$M=-\frac{q}{p}=-\left(\frac{16.7}{25}\right)=-0.668$$

The image is smaller than the object because $|M|<1$, and inverted because *M* is negative

Q3) -- [30 marks]

Derive an expression for the lenses equation.

Answer

Consider a lens having an index of refraction *n* and two spherical surfaces of radii of curvature R_1 and R_2, as in Fig. (1). An object is placed at point O at a distance p_1 in front of surface 1. For this example, p_1 has been chosen so as to produce a virtual image I_1 to the left lens. This image is then used as the object for surface 2, which results in a real image I_2.

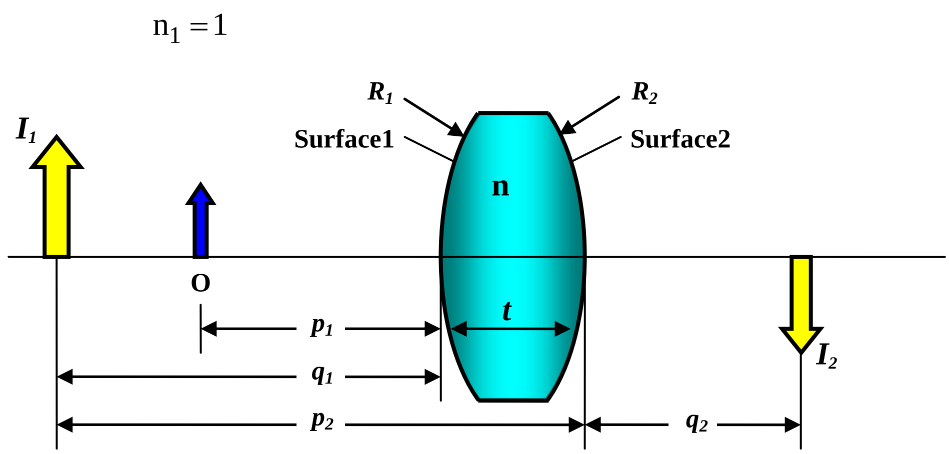

Fig. (1): To locate the image of a lens, the image at I_1 formed by the first surface is used as the object for the second surface. The final image is at I_2

Using Eq. (??) and assuming $n_1 = 1$ because the lens is surrounded by air, we find that the image formed by surface 1 satisfies the equation

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}. \tag{1}$$

Now we apply Eq. (??) to surface 2, taking $n_1 = n$ and $n_2 = 1$. That is, light approaches surface 2 as if it had come from I_1. Taking p_2 as the object distance and q_2 as the image distance for surface 2 gives

$$\frac{1}{p_2} + \frac{n}{q_2} = \frac{1-n}{R_2}. \tag{2}$$

But $p \ = -q_1 + t$, where *t* is the thickness of the lens. (Remember q_1 is a negative number and p_2 must be positive by our sign convention.) For a thin lens, we can neglect *t*. In this approximation and from Fig. (1), we see that $p_2 = -q_1$. Hence, Eq. (2) becomes

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}. \tag{3}$$

Adding Eqs. (1) and (3), we find that

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{4}$$

For the thin lens, we can omit the subscripts on p_1 and q_2 in Eq. (4) and call the object distance *p* and the image distance *q*, as in Fig. (2). Hence, we can write Eq. (4) in the form

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{5}$$

This equation relates the image distance *q* of the image formed by a thin lens to the object distance *p* and to the thin lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is small relative to R_1 and R_2.

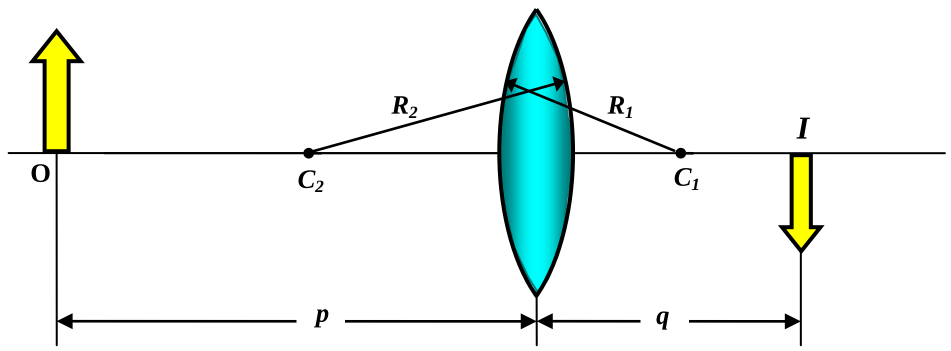

Fig. (2): The biconvex lens.

We now define the focal length *f* of a thin lens as the image distance that corresponds to an infinite object distance, as we did with mirrors. According to this definition and from Eq. (5), we see that as $p \to \infty$, $q \to f$; therefore, the inverse of the focal length for a thin lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \qquad (6)$$

This equation is called the ***lens makers' equation*** because it enables *f* to be calculated from the known properties of the lens. It can be used to determine the values of R_1 and R_2 needed for a given index of refraction and desired focal length. Using Eq. (6), we can write Eq. (5) in an alternate form identical to Eq. (??) for mirrors:

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.}$$