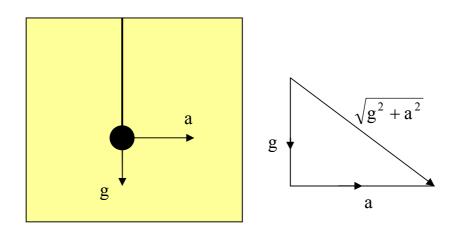
1. A car move by an acceleration a, find the period for a simple pendulum hanging in the car.
 [25]

------ Solution ------



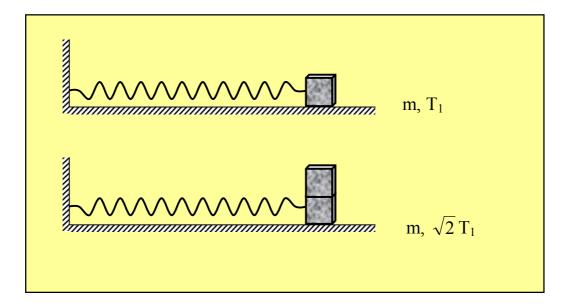
The resultant acceleration is

$$R = \sqrt{g^2 + a^2}$$

The periodic time is

$$T = \sqrt{\frac{\ell}{R}} = \sqrt{\frac{\ell}{\sqrt{g^2 + a^2}}}$$

2. An object of mass m is hung from a spring and set into oscillation. The period of the oscillation is measured and recorded as T. The mass m is removed and replaced with an object of mass 2m. Find the period of motion when this object is set into oscillation. [20]



The period of the pendulum is

$$T_1 = \frac{2\pi}{\omega_1} = 2\pi \sqrt{\frac{m}{k}}$$

If m is replaced by 2m then the new period is

$$T_2 = \frac{2\pi}{\omega_2} = 2\pi \sqrt{\frac{2m}{k}} = \sqrt{2}T_1$$

Differentiating this equation twice with respect to coordinate, x, we get

$$\frac{dy}{dx} = be^{b(x-vt)}$$

$$\frac{d^2y}{dx^2} = b^2 e^{b(x-vt)}$$

$$\frac{d^2y}{dx^2} = -b^2 y$$
(2)

Differentiating Eq. (1) twice with respect to time, t, we get

$$\frac{dy}{dt} = -b\upsilon e^{b(x-\upsilon t)}$$
$$\frac{d^2 y}{dt^2} = -b^2 \upsilon^2 e^{b(x-\upsilon t)}$$
$$\frac{d^2 y}{dt^2} = -b^2 \upsilon^2 y$$

Substituting from (2) and (3) in the wave equation

$$\frac{\mathrm{d}^2 \mathrm{y}}{\mathrm{dt}^2} = \mathrm{v}^2 \, \frac{\mathrm{dy}^2}{\mathrm{dx}^2} \tag{3}$$

We get

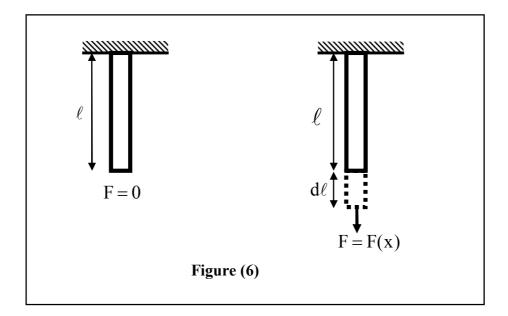
$$-b^2\upsilon^2 y = -b^2\upsilon^2 y$$

So the function satisfying the wave equation and it describes a wave;.

**4.** Show that the work done per unit volume in straining a body is equalto  $\frac{1}{2}$  (Stress × Strain).

------ Solution ------

When a body is deformed by the application of external forces, the body gets strained. The work done is stored in the body in the form of energy and is called the energy of strain. Consider a wire of length  $\ell$ , area cross section A and Young's modulus of elasticity Y, see Fig. (6).



Let  $d\ell$  be the increase in length when a stretching force F is applied. Therefore, work done W

$$W = \int dW = \int_{0}^{d\ell} F(x) dx$$

But F = k x so

W = 
$$\int_{0}^{d\ell} k x \, dx = \frac{1}{2} k x^{2} \Big|_{0}^{d\ell}$$
  
=  $\frac{1}{2} k (d\ell)^{2} = \frac{1}{2} F(d\ell)$ 

Work done per unit volume,

$$w = \frac{W}{V} = \frac{W}{A\ell}$$
$$w = \frac{F(d\ell)}{2A\ell} = \frac{1}{2} \times \frac{F}{A} \times \frac{d\ell}{\ell}$$
$$w = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$