Faculty of Science,
Phys. Department.
Electric circuits

Time: Two Hour
Final Exam
4^{th} Year

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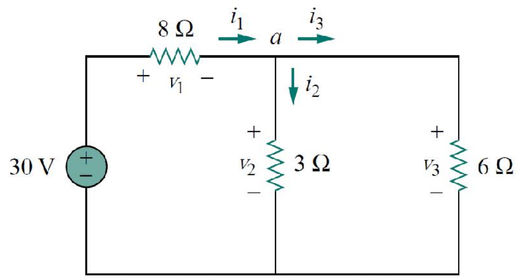
1. Find the currents and voltages in the circuit.



We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \qquad (2.8.1)$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \qquad (2.8.2)$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \qquad (2.8.3)$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Longrightarrow \quad v_3 = v_2 \qquad (2.8.4)$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \quad \Longrightarrow \quad i_3 = \frac{i_2}{2} \qquad (2.8.5)$$

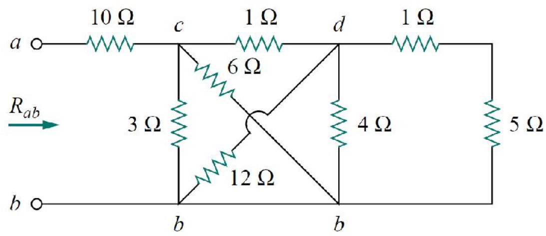
Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V}, \quad v_2 = 6 \text{ V}, \quad v_3 = 6 \text{ V}$$

2. Calculate the equivalent resistance R_{ab} in the circuit



The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad (2.10.1)$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \qquad (2.10.2)$$

With these three combinations, we can replace the circuit in Fig. 2.37 wit that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, a calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in serie with the 1-Ω resistance to give a combined resistance of $1\ \Omega + 2\ \Omega = 3$ Ω

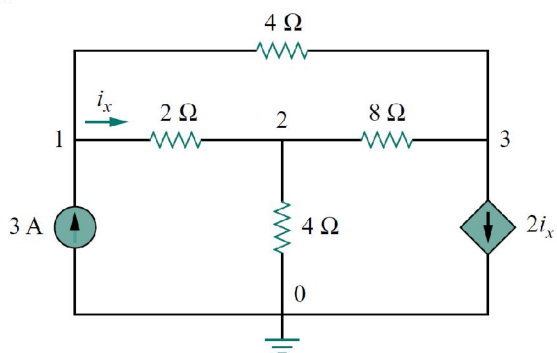
Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

3. Determine the voltages at the nodes in the circuit.



At node 1,

$$3 = i_1 + i_x \quad \Longrightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \qquad (3.2.1)$$

At node 2,

$$i_x = i_2 + i_3 \quad \Longrightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \qquad (3.2.2)$$

At node 3,

$$i_1 + i_2 = 2i_x \quad \Longrightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3.2.3)$$

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4. For the circuit in Fig. 3, find i_1 to i_4 using mesh analysis.

KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \qquad (3.7.1)$$

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5 \qquad (3.7.2)$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3i_o$$

But $i_o = -i_4$, hence,

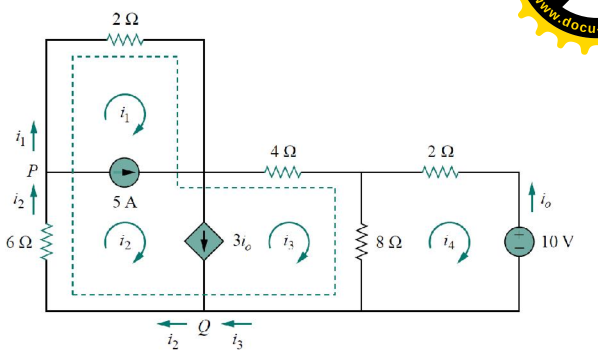
$$i_2 = i_3 - 3i_4 \qquad (3.7.3)$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

$$5i_4 - 4i_3 = -5 \qquad (3.7.4)$$



From Eqs. (3.7.1) to (3.7.4),

$$i_1 = -7.5 \text{ A}, \qquad i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}, \qquad i_4 = 2.143 \text{ A}$$

5. By inspection, write the mesh-current equations for the circuit.

We have five meshes, so the resistance matrix is 5 by 5. The diagonal terms, in ohms, are:

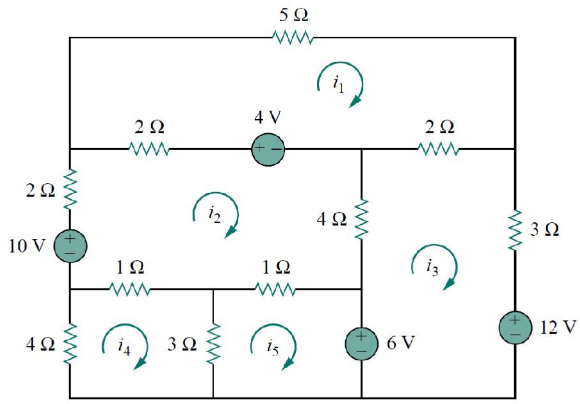
$$R_{11} = 5 + 2 + 2 = 9, \qquad R_{22} = 2 + 4 + 1 + 1 + 2 = 10$$
$$R_{33} = 2 + 3 + 4 = 9, \qquad R_{44} = 1 + 3 + 4 = 8, \qquad R_{55} = 1 + 3 = 4$$

The off-diagonal terms are:

$$R_{12} = -2, \qquad R_{13} = -2, \qquad R_{14} = 0 = R_{15}$$
$$R_{21} = -2, \qquad R_{23} = -4, \qquad R_{24} = -1, \qquad R_{25} = -1$$
$$R_{31} = -2, \qquad R_{32} = -4, \qquad R_{34} = 0 = R_{35}$$
$$R_{41} = 0, \qquad R_{42} = -1, \qquad R_{43} = 0, \qquad R_{45} = -3$$
$$R_{51} = 0, \qquad R_{52} = -1, \qquad R_{53} = 0, \qquad R_{54} = -3$$

The input voltage vector **v** has the following terms in volts:

$$v_1 = 4, \qquad v_2 = 10 - 4 = 6$$
$$v_3 = -12 + 6 = -6, \qquad v_4 = 0, \qquad v_5 = -6$$



Thus the mesh-current equations are:

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

From this, we can obtain mesh currents i_1, i_2, i_3, i_4, and i_5.

6. Find the Thevenin equivalent of the circuit.

Applying mesh analysis to loop 1 in the circuit in Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \qquad (4.9.1)$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \qquad (4.9.2)$$
$$6(i_3 - i_2) + 2i_3 + 1 = 0 \qquad (4.9.3)$$

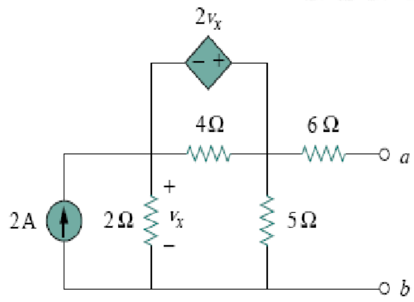
Solving these equations gives

$$i_3 = -\frac{1}{6} \text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{Th} = \frac{1 \text{ V}}{i_o} = 6 \ \Omega$$

To get V_{Th}, we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get



$$i_1 = 5 \qquad (4.9.4)$$
$$-2v_x + 2(i_3 - i_2) = 0 \quad \Longrightarrow \quad v_x = i_3 - i_2 \qquad (4.9.5)$$
$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

or

$$12i_2 - 4i_1 - 2i_3 = 0 \qquad (4.9.6)$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{Th} = v_{oc} = 6i_2 = 20 \text{ V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

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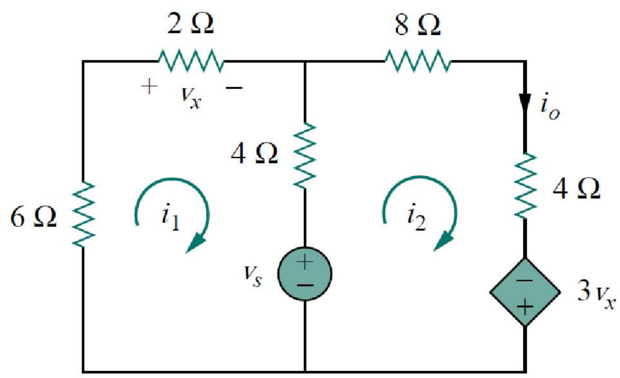
7. For the circuit, find i_o when v_s = 12 V and v_s = 24 V.



Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Longrightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Longrightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

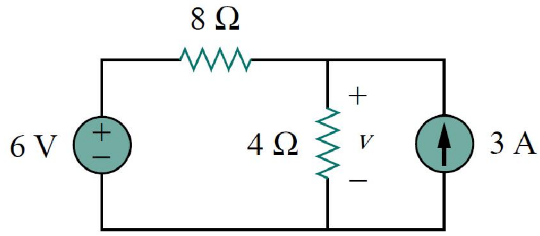
$$i_o = i_2 = \frac{12}{76} \text{ A}$$

When $v_s = 24$ V,

$$i_o = i_2 = \frac{24}{76} \text{ A}$$

showing that when the source value is doubled, i_o doubles.

8. Use the superposition theorem to find v in the circuit.



Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and

$$12i_1 - 6 = 0 \quad \Longrightarrow \quad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

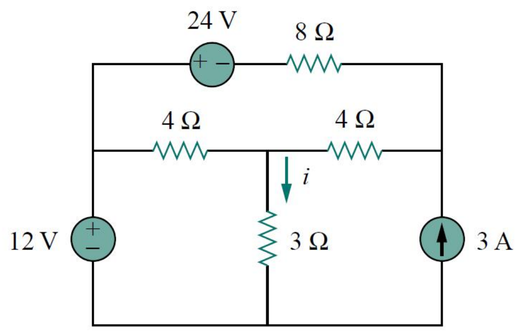
Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

9. For the circuit, use the superposition theorem to find i.



In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{12}{6} = 2 \text{ A}$$

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis,

$$16i_a - 4i_b + 24 = 0 \quad \Longrightarrow \quad 4i_a - i_b = -6 \qquad (4.5.1)$$

$$7i_b - 4i_a = 0 \quad \Longrightarrow \quad i_a = \frac{7}{4}i_b \qquad (4.5.2)$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis,

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Longrightarrow \quad 24 = 3v_2 - 2v_1 \qquad (4.5.3)$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Longrightarrow \quad v_2 = \frac{10}{3}v_1 \qquad (4.5.4)$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

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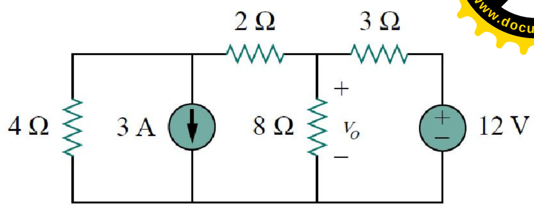
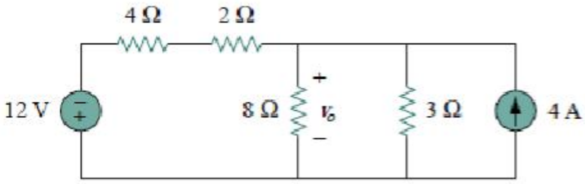
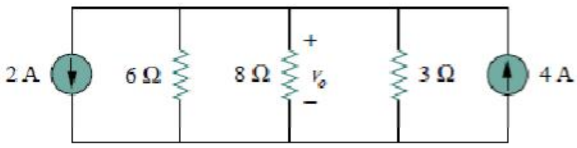
10. Use source transformation to find v_o in the circuit



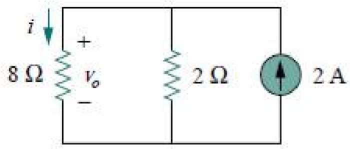
Solution:



(a)



(b)



(c)

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2+8}(2) = 0.4$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

………………………………………………………………………… ***With my best wishes&***